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Focusing femtosecond X-ray free-electron laser pulses by refractive lenses

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The possibility of using a parabolic refractive lens with initial X-ray free-electron laser (XFEL) pulses, *i.e.* without a monochromator, is analysed. It is assumed that the measurement time is longer than 0.3 fs, which is the time duration of a coherent pulse (spike). In this case one has to calculate the

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XFEL

X-Ray Free-Electron Laser

The European X-Ray Free-Electron Laser

Technical design report

http://www.xfel.eu/sites/site_xfel-gmbh/content/e63617/e79991/e68669/european-xfel-tdr_eng.pdf

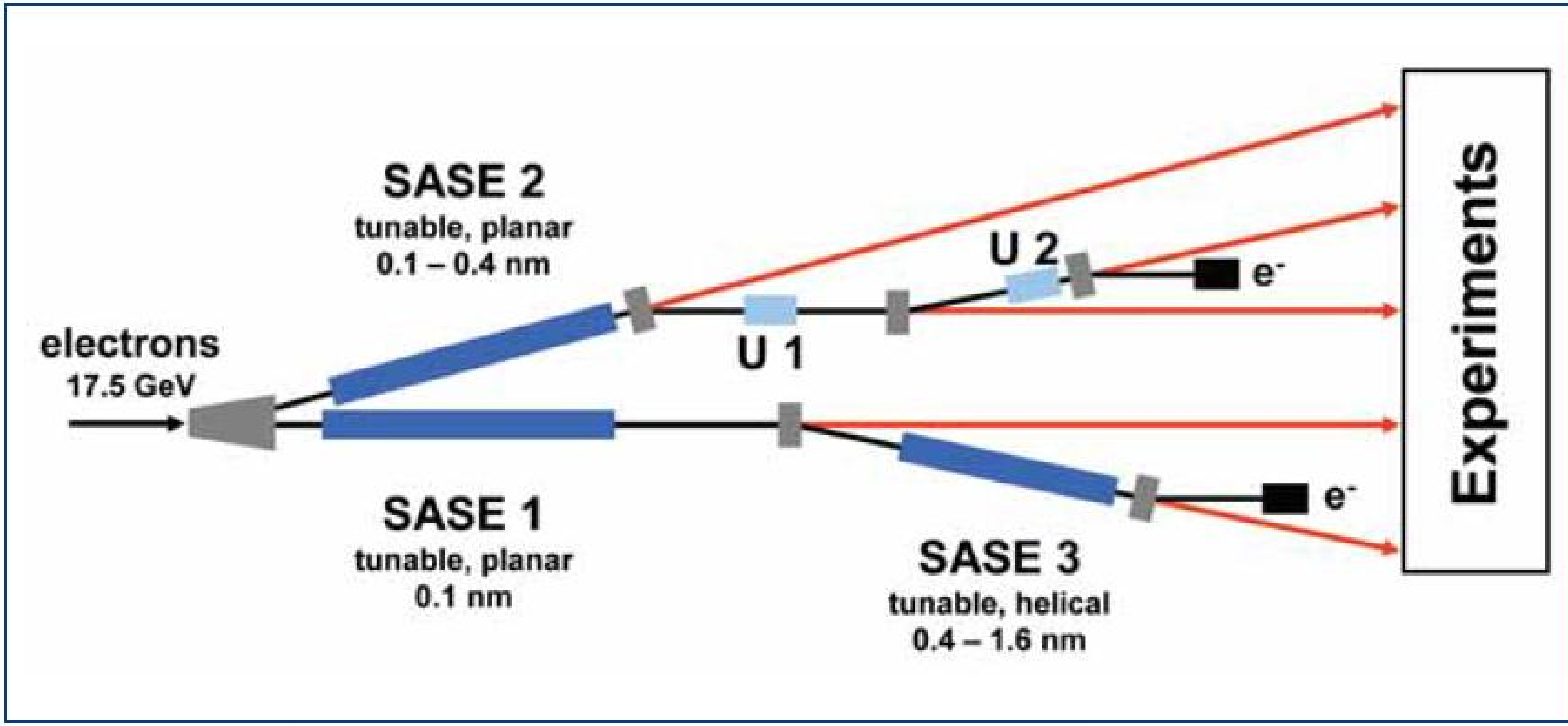


Figure 4.2 *Schematic view of the branching of electron (black) and photon (red) beamlines through the different SASE and spontaneous emission undulators. Electron beamlines terminate in the two beam dumps and photon beamlines in the experimental hall.*

	Photon energy [keV]	Polarisation	Tunability	Gap variation
SASE 1	12.4	Linear	No	Yes
SASE 2	3.1 – 12.4	Linear	Yes	Yes
SASE 3	0.8 – 3.1 (0.25 – 1.0)*	Circular/Linear	Yes	Yes
U1, U2	20 – 100	Linear	Yes	Yes

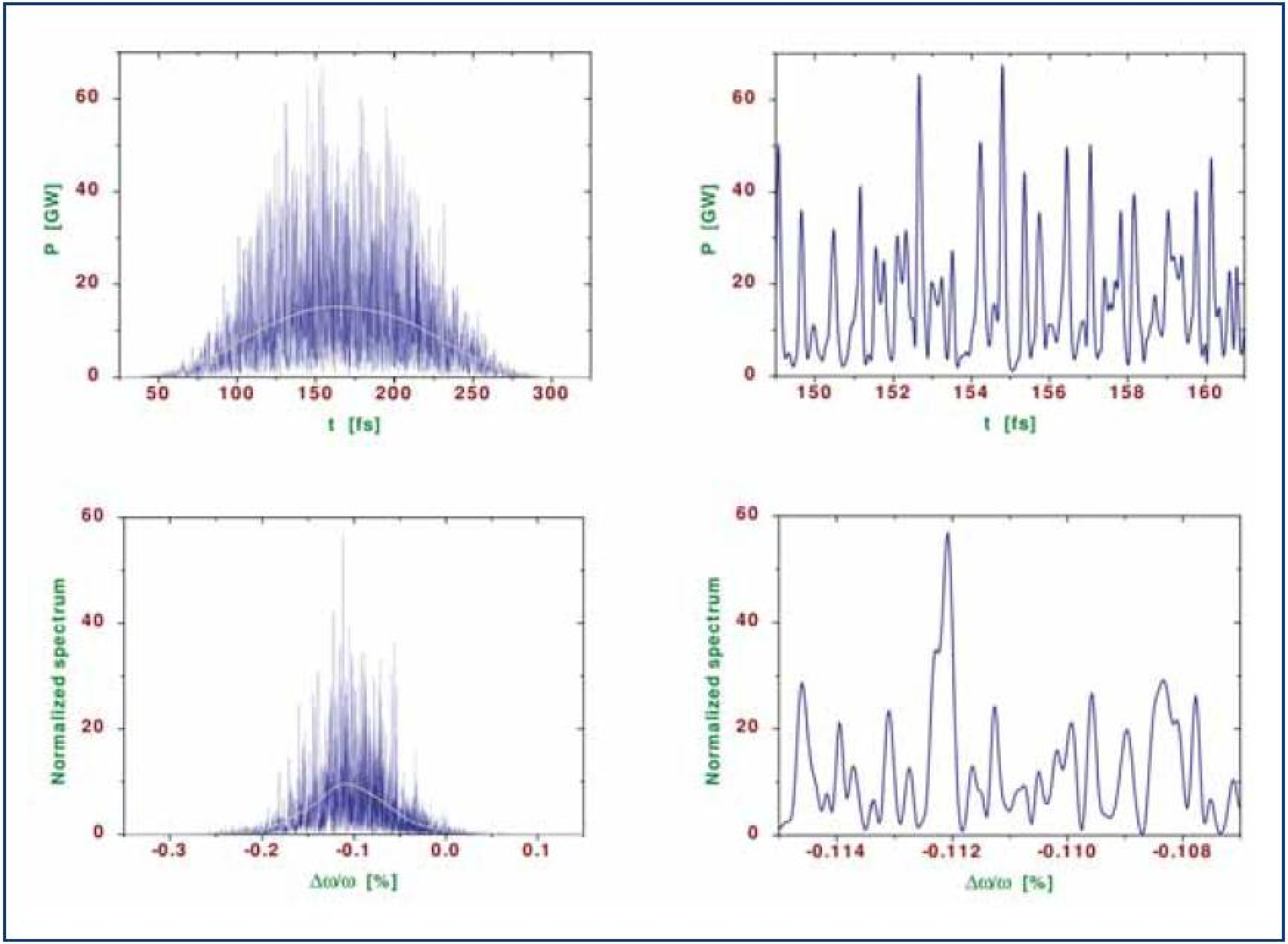


Figure 5.2.4 *Temporal (top) and spectral (bottom) structure for 12.4 keV XFEL radiation from SASE 1. Smooth lines indicate averaged profiles. Right side plots show enlarged view of the left plots. The magnetic undulator length is 130 m.*

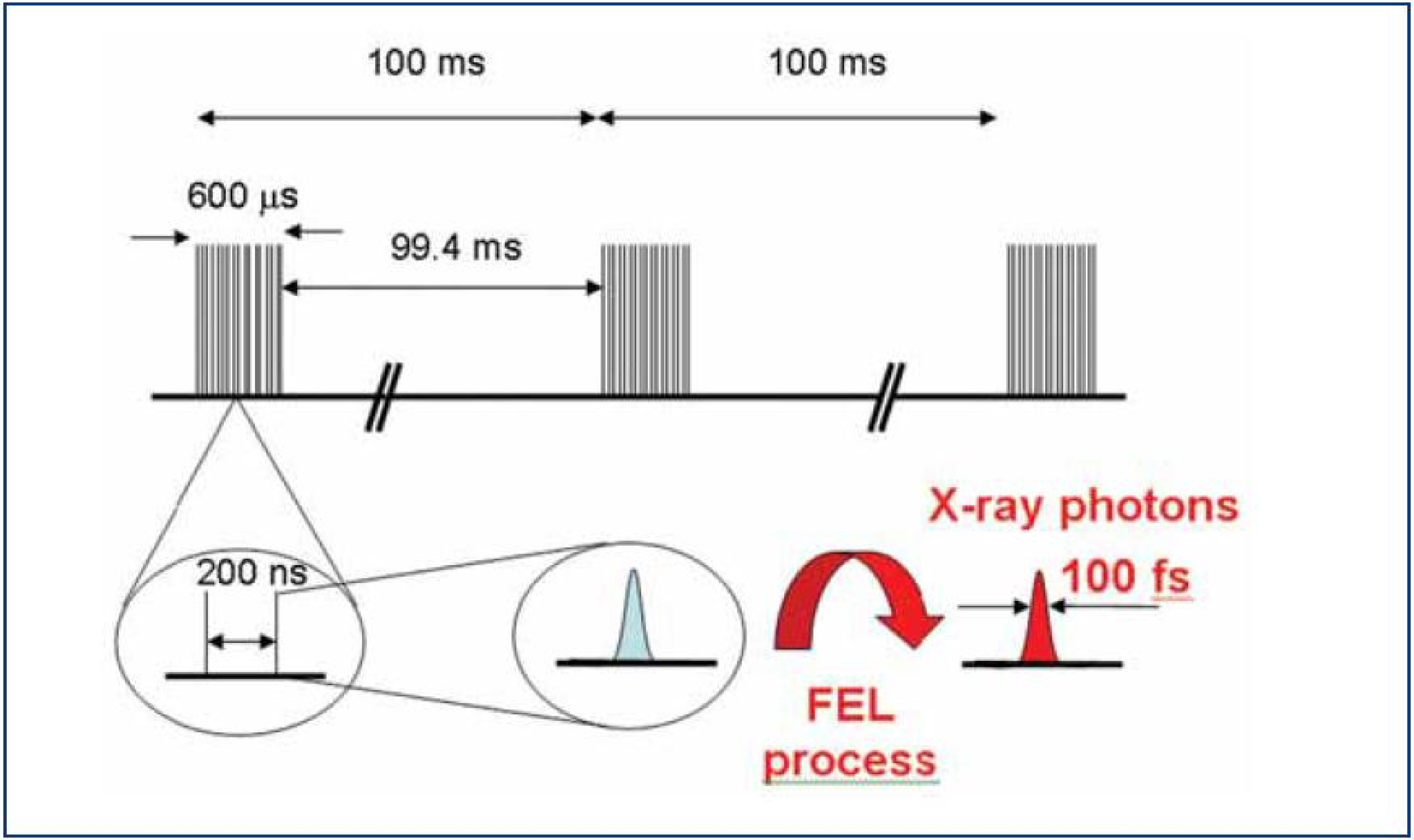


Figure 6.1.1 *Electron bunch time pattern with 10 Hz repetition rate and up to 3,000 bunches in a 0.6 ms long bunch train. The separation of electron bunches within a train is 200 ns for full loading. The duration of electron bunches is ~200 fs and the non-linear FEL process reduces the duration of the photon pulses to ~100 fs.*

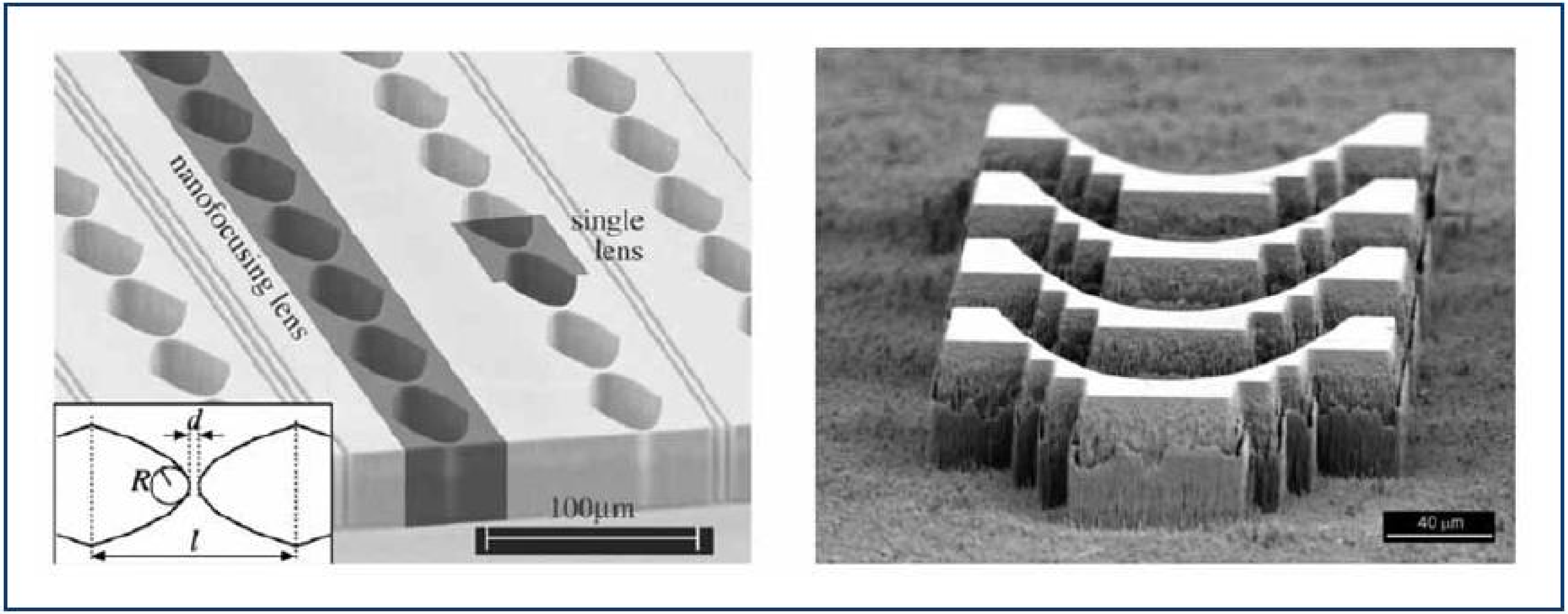


Figure 6.2.7 *Left: NFLs made by electron beam lithography and reactive ion etching of Silicon* **[6-19]**. *Right: Diamond planar refractive lenses fabricated using a similar technique* **[6-23]**.

Chromaticity

Due to the quadratic dependence of the refractive index for hard x-rays on the x-ray wavelength, refractive lenses are strongly chromatic. Optimal performance can only be reached for monochromatic x-rays, i. e. $\Delta E/E<10^{-4}$. The natural width of the SASE radiation ($\Delta E/E \sim 10^{-3}$) will be focused to an interval along the optical axis that is about one order of magnitude larger than the depth of focus, effectively broadening the focus to several 100 nm. If this is not acceptable for a given experiment, a reduction of the bandwidth using a crystal monochromator may be required.

Time dependence can be calculated by means of frequency dependence

$$E(x, z, t) = \int \frac{\mathrm{d}\omega}{2\pi} \exp(-i\omega t)\, E(x, z, \omega),$$

Parseval's theorem, that means conservation of integral intensity

$$\langle I(x, z)\rangle = \int \mathrm{d}t\, \left|E(x, z, t)\right|^2 = \int \frac{\mathrm{d}\omega}{2\pi} \left|E(x, z, \omega)\right|^2.$$

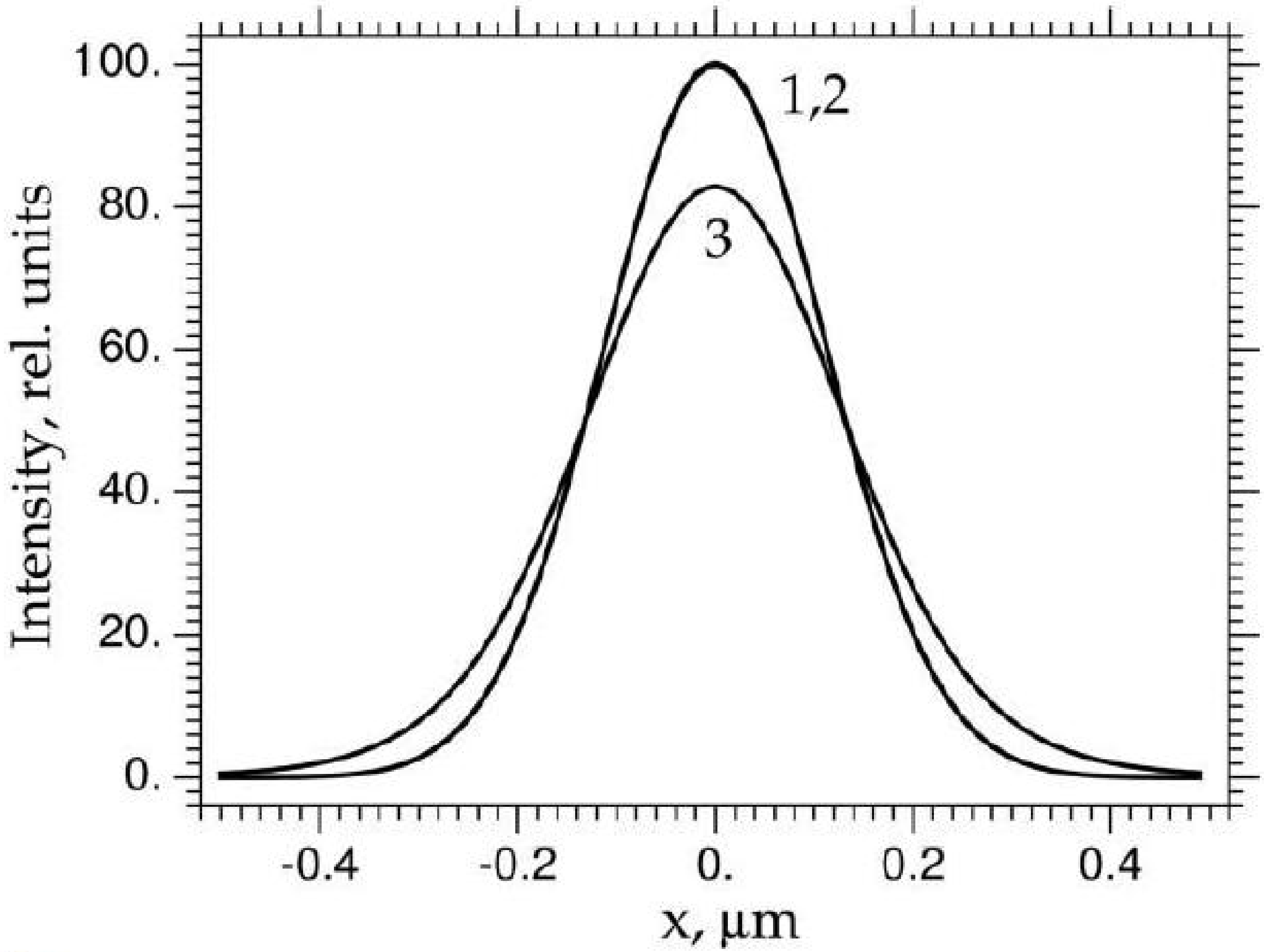


Figure 5

Relative intensity profiles at the focus for a parabolic refractive lens made from Si, with the curvature radius $R = 1$ μm, for $E = 12.4$ keV, and $\Delta E/E = 0$ (curve 1), 10^{-3} (curve 2) and 10^{-2} (curve 3). Curves 1 and 2 completely coincide with each other and have a maximum of 100. See text for more details.

The figure is the same for the lens with R = 0.01 μm but with 10 times smaller x-scale.

$$\langle I(x, z, E)\rangle = \int \mathrm{d}E_1 \frac{z_t}{|b(z, E_c)|} \exp\left[-\frac{x^2}{2\sigma^2(z, E_c)}\right] G(E_1, \Delta E), \tag{42}$$

where the parameters b, σ and z_t are described by (27), (28), $E_c = E + E_1 = \hbar\omega$, and

$$G(E_1, \Delta E) = \frac{1}{(2\pi)^{1/2}\sigma_E} \exp\left(-\frac{E_1^2}{2\sigma_E^2}\right), \quad \sigma_E = \Delta E/e_2. \tag{43}$$

We note that $\lambda = hc/E_c$ where h is Planck's constant, $hc = 1.23984$ keV nm.

Let us consider now an arbitrary lens of our lens system (the first as a particular case) and introduce the most general form for the Gaussian wave as follows (Kohn, 2003),

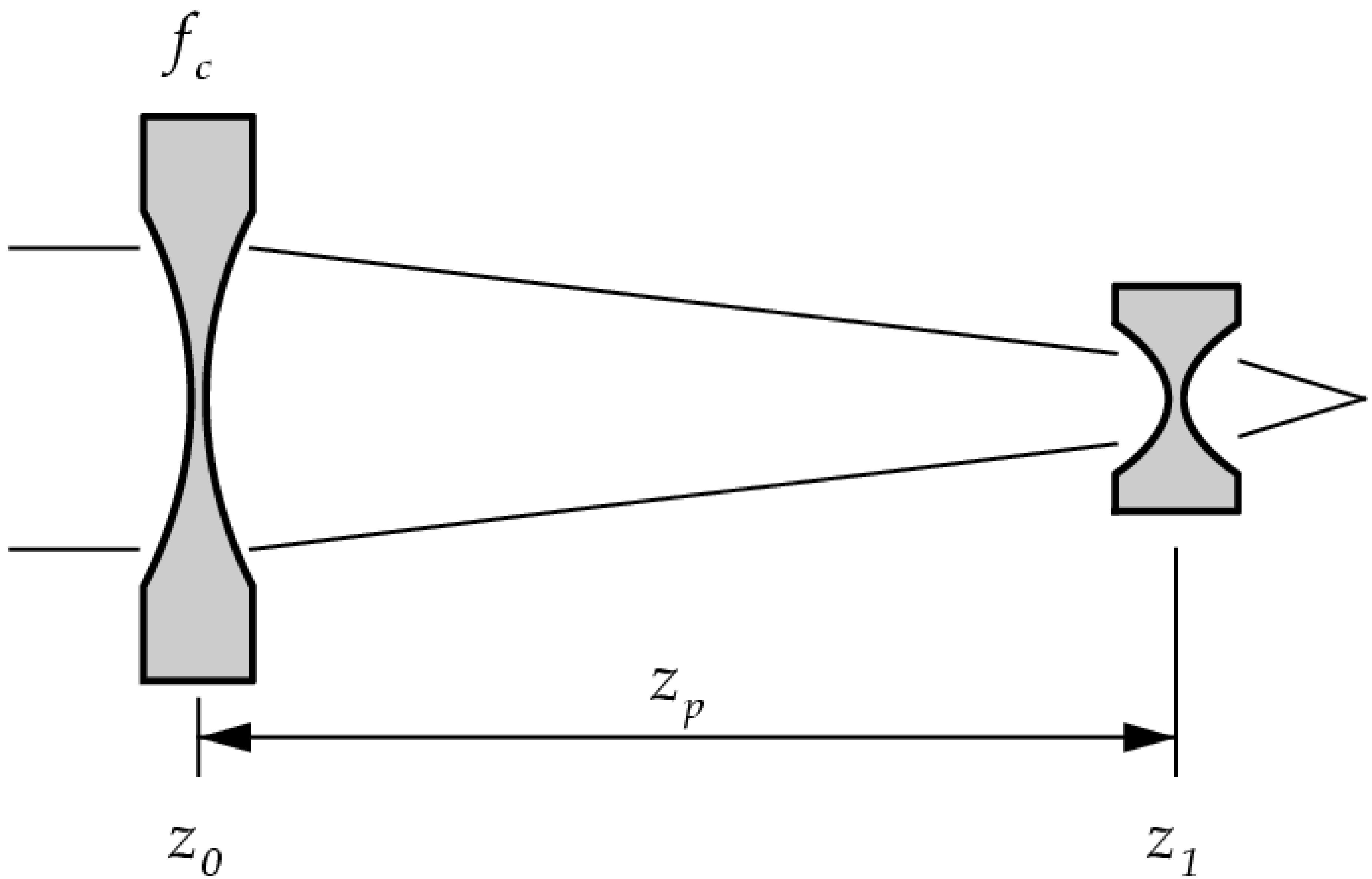
$$E_0(x, x_0) = T(x, a_0)\, P(x - x_0, b_0)\, T(x_0, c_0), \tag{22}$$

where a_0, b_0, c_0 are the complex parameters, the index 0 means the position on the optical axis (z_0) and

$$\begin{aligned} T(x, a) &= \exp\left(-i\pi \frac{x^2}{\lambda a}\right), \\ P(x, b) &= (i\lambda b)^{-1/2} \exp\left(i\pi \frac{x^2}{\lambda b}\right). \end{aligned} \tag{23}$$

We want to formulate a theorem that the form (22) is conserved on all paths from the first lens to the detector, and only the complex parameters change their values. This change takes the form of relations which can simplify both analytical and numerical calculations of complex optical systems.

The task: propagation of Gaussian wave through one lens and the distance to another lens



The result of the calculation can be written in the same form,

$$E_1(x, x_0) = T(x, a_1)\, P(x - x_0, b_1)\, T(x_0, c_1), \tag{25}$$

where a_1, b_1, c_1 are the new complex parameters and the index 1 means a position on the optical axis (z_1). The new parameters a_1, b_1, c_1 of the wavefunction at a distance z_p behind the lens can be calculated from the initial parameters a_0, b_0, c_0 of the wave (22) by means of the formulae

$$\begin{aligned} a_1 &= d(b_1/b_0), \qquad & b_1 &= b_0 + z_p - (z_p b_0/d), \\ c_1 &= c_0/(1 + z_p c_0/b_1 d), \qquad & d &= a_0/(1 + a_0/f_c). \end{aligned} \tag{26}$$

The derivation is shown in Appendix *A*. The relations (26) allow one to calculate an optical system consisting of an arbitrary number of refractive parabolic lenses placed with

The model of incident wave

the model which was proposed for the first time by Kohn *et al.* (2009). In this model the wavefunction in front of the first lens is described by the same function $P(x - x_0, z_{0c})$, but with the complex longitudinal coordinate $z_{0c} = z_0 - i\sigma$ under the condition $\sigma \ll z_0$. If α_0 is the angular divergence of the incident-beam intensity (FWHM), then it is easy to show that

$$\sigma = \lambda e_1^2/\alpha_0^2, \tag{21}$$

where e_1 is determined by (18). Substituting the parameters of XFEL, namely $\lambda = 0.1$ nm, $\alpha_0 = 1$ μrad, we obtain $\sigma = 44$ m. So the condition $z_0 > \sigma$ can be fulfilled on an XFEL source where z_0 is greater than 500 m. Under the specified condition the angular divergence of the beam is independent of distance, The task "focusing the plane wave by two-lens system" has now analytical solution, it is new

Neglecting absorption of X-rays in the lenses, we obtain immediately that the focus distance $z_f = \mathrm{Re}(F_c)$ of the two-lens system, counted from the second lens, satisfies the relation

$$\frac{1}{z_f} = \frac{1}{f_1 - z_1} + \frac{1}{f_2}. \tag{38}$$

If $z_1 < f_1$ then $z_f < f_2$.

Two lenses cannot make the beam smaller

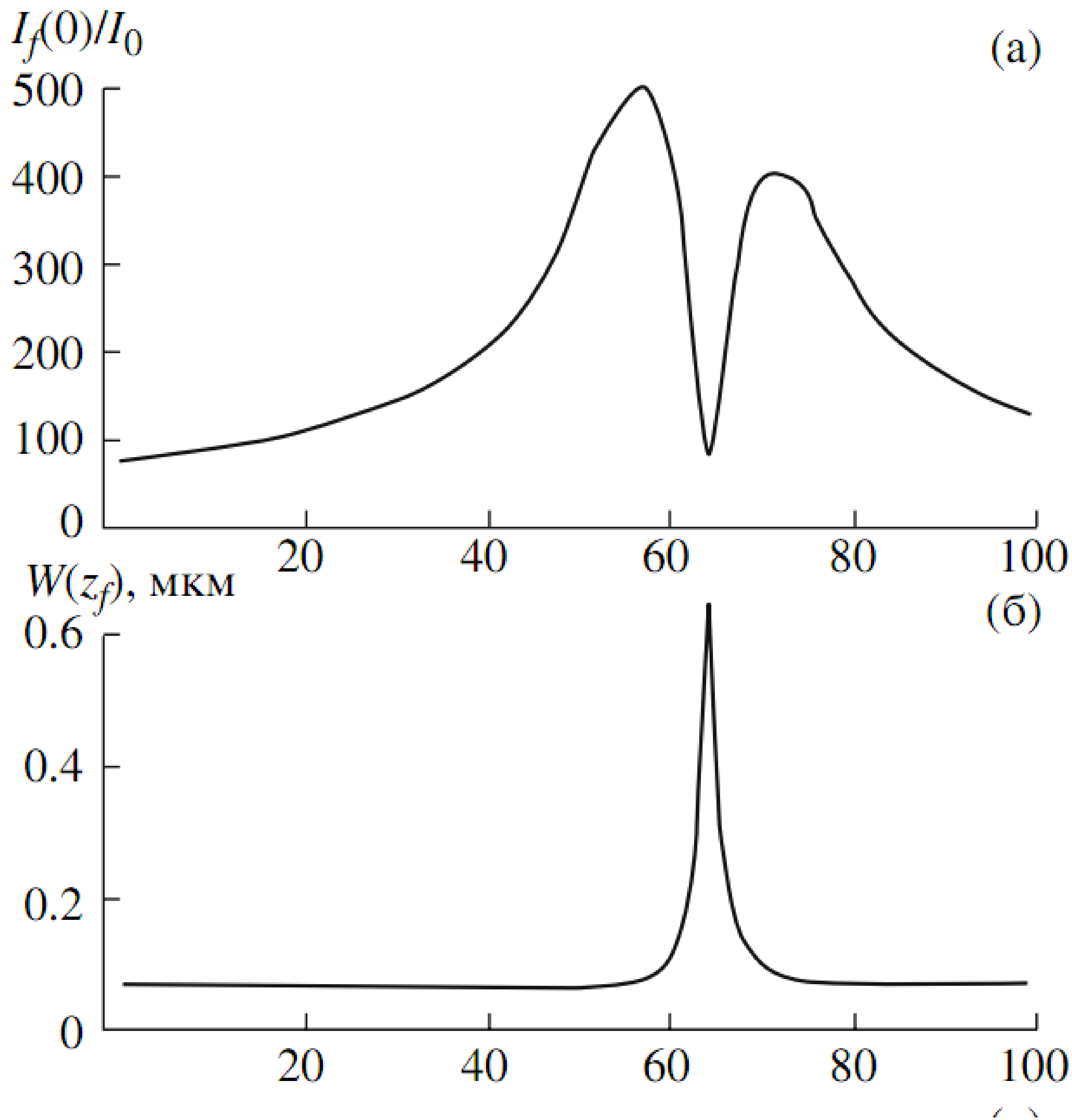
The width of the beam at the focus can be obtained from the imaginary part of F_c calculated in the linear over γ approximation. We can write the FWHM of the beam $w(z_f)$ at the focus in the form

$$w(z_f) = w_2(f_2)\frac{(p+h^2)^{1/2}}{|p+h|}, \tag{39}$$

where

$$w_2(f_2) = e_1(\lambda f_2 \gamma)^{1/2}, \quad p = \frac{f_2}{f_1}, \quad h = 1 - \frac{z_1}{f_1} \tag{40}$$

Here $w_2(f_2)$ is the beam width at the focus in the case of focusing a plane wave by only second lens. The calculation is direct and is shown in Appendix B.



$I_f(0)/I_0$
(а)
500
400
300
200
100
0
20
40
60
80
100
$W(z_f)$, мкм
(б)
0.6
0.4
0.2
0
20
40
60
80
100

БЛАГОДАРЮ

ЗА

ВНИМАНИЕ